Computational Continuations

John Quigley
www.jquigley.com
jquigley@jquigley.com

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The ‘rest of the computation’ means control state, or the data structures and code needed to complete a computation.

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Most languages have facilities for manipulating the continuation of a computation step.

Early imperative languages provided the GOTO – or setjmp(3) in C – which would force the computation to continue at some designated label.

In the 1970’s, additional control patterns were added like function returns, loop exits and iteration breaks.

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Availability

1 Parrot: Continuation
2 Ruby: callcc
3 Scheme: call-with-current-continuation, or call/cc
4 Smalltalk: Continuation currentDo:

In any language which supports closures, it is possible to manually implement call/cc!

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def foo(x):
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The core idea of continuations is to make this behavior explicit by adding a continuation argument.

Instead of ‘returning’ the value, the function ‘continues’ with the value by giving it as an argument to the continuation.

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With this view, a function never ‘returns’ – instead it ‘continues.’

And it is for this reason, continuations have sometimes been described as gotos with arguments.

This idea is the basis of CPS, or Continuation Passing Style:

- Function signature gets extra ‘continuation’ argument
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We can simulate this with the following definitions:

**simulated primitives**

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def add(x, y, c): c(x+y)
def mul(x, y, c): c(x*y)
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CPS would transform the `baz()` function into:

**cps transformation**

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def baz(x, y, c):
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More On CPS

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Continuations are the functional expression of the GOTO statement, and the same caveats apply.

Continuations can quickly result in code that is difficult to follow: the programmer must maintain the invariants of control and continuations by hand.

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The most basic form of a continuation is a subroutine call.

**Definition:** A generator is a special subroutine that can be used to control loop iteration behavior.

A generator looks like a function, but behaves like an iterator.

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Coroutines add only one new abstract operation: *transfer*.

‘Transfer’ names a coroutine to transfer to, and gives a value to deliver to it.

When A transfers to B, it acts like a generator ‘suspend’.

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Give the pedagogical structure so far, you’re primed to view continuations as enhancements of coroutines.

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Subroutines

Let’s look at Python.

When Python makes a call, it allocates a frame object. When a subroutine returns, it decrefs the frame and it goes away.

Attached to that frame:

- locals, or a map of name:object bindings
- evaluation stack for holding temps and dynamic block-nesting info
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Generators are a trivial extension on what Python does with subroutines.

When a generator suspends, it’s just like a return, except we decline to decref the frame. That’s it!

The locals, and where we are in the computation, aren’t thrown away.

A ‘resume,’ then, consists of restarting the frame at its next bytecode instruction, with the locals and eval stack retained.
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